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THE MONOPSONISTIC FIRM IN AN UNCERTAIN WORLD

by

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b is a random variable with mean $E(b) = \beta$ and density $f(b)$. Further, define $bK = K_1$, and assume $f_{K_1} > 0$ and $f_{K_1' K_1'} < 0$. In all three cases it is assumed that the firm seeks to maximize expected utility from profits; that its attitude toward risk can be described by a von Neumann-Morgenstern cardinal utility function (U) and that the decision concerning the utilization of input is made prior to knowledge of the market situation. Finally, we assume that the input price is described by the function $r = r(K)$.

II. CASE 1

Profits for the firm facing an uncertain price are given by

$$\pi = pq - r(K)\, K \qquad (1)$$

Under the assumption that the firm maximizes the expected utility of profit the first and second order conditions for a maximum are respectively,

$$E\,\{U'(\pi)\ (pf_K - r(K) - r_K K)\ \} = 0 \qquad (2)$$

and

$$B = E\{U''(\pi)\ \ (pf_K - r(K) - r_K K)^2 + U'\ (\pi)\ \ (pf_{KK} - r_{KK}K - 2r_K)\} < 0 \qquad (3)$$

Here, as elsewhere, we shall assume that the second order condition holds.

To demonstrate that the firm will demand less of K and, therefore, produce less of q in the case of uncertainty, we follow Sandmo (5) and compare the uncertain case with the case where p is known to take its mean value μ with certainty. After a slight manipulation (2) can be rewritten as

$$\mu f_K = r(K) + r_K K - \frac{f_K \mathrm{Cov}(U'(\pi), p)}{E\{U'(\pi)\}} \qquad (4)$$

Now differentiation of $U'(\pi)$ with respect to p obtains $U''(\pi)\, q$ which will be less than zero if the firm is risk averse, equal to zero if the firm is risk neutral and less than zero if the firm is a risk preferrer. Consequently, the risk averse (preferring) firm will employ less (more) of the factor in an uncertain world, and hence, will produce less (more) in an uncertain world, while the risk neutral firm equates the expected value of its marginal product to marginal outlay.

To examine the effect of an increase in risk as defined by an increased variability of the density function of price in terms of a mean preserving spread, define $p^*=p\gamma+\theta$ where γ and θ are shift parameters which initially equal one and zero respectively. Noting that a mean preserving spread implies $\frac{d\theta}{d\gamma} = -\mu$, and substituting p^* for p in (2) and differentiating with respect (remembering that $\gamma = 1$ and $\theta = 0$ originally) to γ obtains

$$B\frac{\partial K}{\partial \gamma} = -E\{qU''(\pi)\ (p-\mu)\ (pf_K-r_K K-r(K)) + U'(\pi)\ (p-\mu)\ f_K\} \qquad (5)$$

To proceed note that following Sandmo(5) profit can be written as $\pi= E(\pi) + (p-\mu)q$ and further that $\pi > E(\pi)$ for $p > \mu$. Now a risk averse firm is characterized by $U''(\pi) < 0$ which umplies $U'(\pi) < U'\{E(\pi)\}$ for $p > \mu$. Multiplying both sides of this inequality by $f_K(p-\mu)$ and taking expectations, remembering that $E(\pi)$ is a fixed number, obtains $E\{U'(\pi)f_K(p-\mu)\} < 0$.[1/] This result and the fact that by(3) $B < 0$ establishes that the sign of $\frac{\partial K}{\partial \gamma}$ depends upon the first term on the right hand side of (5). Following Batra and Ullah (2) it will be demonstrated that for a firm characterized by decreasing absolute risk aversion, $\frac{\partial K}{\partial \gamma} < 0$, i.e., the right side of (5) is positive. To proceed, recall that a firm's attitude toward risk can be defined by its absolute risk aversion function $R_a(\pi) = -\frac{U''(\pi)}{U'(\pi)}$. Now rewriting the first term in the braces on the right hand side of (5) we have

$$-E\left\{qU''(\pi)\ (pf_K-r_K K-r(K))\ \left[p - \frac{r(K)+r_K K}{f_K} + \frac{r(K)+r_K K}{f_K} - \mu\right]\right\}$$

which in turn can be rewritten

$$-E\left\{qU''(\pi)\ \frac{(pf_K-r_K K-r(K)\ )^2}{f_K}\right\} -E\left\{U''(\pi)\ (pf_K-r_K K-r(K)\ )\right.$$

$$\left. X\ \ \frac{r(K)\ +\ r_K K}{f_K} - \mu\right\}$$

Now under risk aversion the first term will be positive and we have already demonstrated that $\frac{r(K) + r_K K}{f_K} - \mu < 0$ for the risk averse firm. Therefore, we must sign $E\{U''(\pi)\ (pf_K - r_K K - r(K)\ )\}$. Again, following Sandmo (5) let $\bar{\pi}$ be the profit level when $pf_K = r(K) + r_K K$. Now if $R_a(\pi) > 0$ and $R_a'(\pi)$ [illegible] then it follows that

$$- \frac{U''(\pi)}{U'(\pi)} = R_a(\pi) \underset{-}{<} Ra(\bar{\pi}) \qquad pf_K \underset{-}{>} r_K K + r(K)$$

where $Ra(\bar{\pi})$ is a given number. Now multiplying both sides by $-U'(\pi)\ (pf_K - r_K K - r(K))$ and taking expectations obtains

$$E\{U''(\pi)\ (pf_K - r_K K - r(K)\} \underset{-}{>} R_a(\bar{\pi})\ E\ \{U'(\pi)\ (pf_K - r_K K - r(K)\ )\ \} = 0$$

by (2). Therefore, the right hand side of (5) is positive, and we may conclude that an increase in γ leads to a decrease in the level of input utilized and consequently to a decline in production.

Turning to the effect of an increase in the expected value of p, rewrite p as p + Θ, substitute this into (2), differentiate with respect to Θ and evaluate the derivatives at $\Theta = 0$ to obtain

$$B \frac{\partial K}{\partial \Theta} = -E\{qU''(\pi)\ (pf_K - r_K K - r(K))\} - f_K E\{U'(\pi)\} \qquad (6)$$

By our earlier arguments it is clear that the right side of (6) is negative so that the effect of an increase in expected price is a greater utilization of K, and hence a larger output.

III. CASE 2

In case 2, the expected utility from profits for the monopsonistic firm can be written

$$E\{U(pq - r(K)K)\} \qquad (7)$$

where p is non-stochastic and $q = af(K)$. To demonstrate that the results obtained for the firm facing an uncertain price apply in this case, it is sufficient to recognize that we can define a new variable $\tilde{p} = \alpha p$ which will have expectation $\alpha p = \tilde{\mu}$ and the proofs presented above will apply directly once we redefine (7) to read

$$E\{U(\tilde{p}f(K)-r(K)K)\} \tag{8}$$

and replace μ with $\tilde{\mu}$ in the above.

Therefore, the monopsonistic firm facing production uncertainty as characterized in this model will hire less in the uncertain case than in its certainty equivalent. An increase in risk defined by a mean preserving spread on f(a) will decrease the level of factor inputs hired. Finally, an increase in the expected value of a will lead to increased factor employment.

IV. CASE 3

The analysis of case 3 is not nearly as clearcut as for the two previous cases since the profit function is no longer a linear function of the random variable. However, it is straightforward to replicate the basic results Ratti and Ullah (4) have found for the competitive firm, i.e., a risk neutral monopsonistic firm will demand less of the factor of production in the uncertain than in the certain case. As before the monopsonistic firm seeks to maximize

$$E\{U(pf(bK)-r(K)K)\} \tag{9}$$

First and second order conditions for a maximum are respectively,

$$E\{U'(\pi)\ (pbf_{K_1}(bK)-r_K K-r(K)\ )\ \} = 0 \tag{10}$$

and

$$B_2 = E\{U''(\pi)\ (pbf_{K_1}-r_K K-r(K)\ )^2 + U'(\pi)\ (pb^2 f_{K_1,K_1} - 2r_K - r_{KK}K)\ \} < 0 \quad (1$$

Now the first order condition, following the previous discussion can be rewritten as

$$pE[bf_{K_1}] = r(K) + r_K K - \frac{p\mathrm{Cov}[U'(\pi), bf_{K_1}]}{E[U'(\pi)]} \tag{12}$$

Now following Ratti and Ullah (4) we note that

$$\frac{\partial U'(\pi)}{\partial b} = pU''(\pi)Kf_{K_1} \quad \text{and} \quad \frac{\partial bf_{k_1}}{\partial b} = f_{K_1}(1+\varepsilon)$$

where $\varepsilon = \frac{f_{K_1,K_1}K_1}{f_{K_1}}$, i.e. the elasticity of the marginal product curve. If this elasticity is greater than -1, then Cov $(U'(\pi), bf_{K_1})$ will take the same sign as $U''(\pi)$ 2/. Assuming this is the case implies that the risk preferring firm will demand more of K than the risk neutral firm while the risk neutral firm will employ K up to the point where the expected value of its marginal product is equal to marginal outlay $(r_K K+r(K))$ for the input.

Turning to the analysis of relative input demand in the certain and uncertain cases note that under conditions of certainty the first order condition for a maximum for the risk neutral firm is

$$p\beta f_{K_1}(\beta K) = r(K)+r_K K \tag{13}$$

and for the uncertainty case the first order condition is

$$pE\{bf_{K_1}\} = r(K) + r_K K \tag{13'}$$

The fact that $bf_{K_1}(bK)$ will be concave in b if ε is nonincreasing in K_1 (See Ratti and Ullah (4)) implies by Jensen's inequality that

$$\beta f_{K_1}(\beta K) > E\{bf_{K_1}(bK)\} \tag{14}$$

which in turn implies that the marginal outlay for the risk neutral firm in the certain case is greater than the marginal outlay for the risk neutral firm in the uncertain case if ε is nonincreasing in K_1. Now if marginal outlay is an increasing function of K, which is insured if $r_{KK} > 0$, then the risk neutral firm will hire more in the certain case than in the uncertain case. 3/

To see the effect of an increase in uncertainty, let $f(b)$ undergo a mean-preserving spread. Define $b^* = \gamma b+\Theta$, then replace b in (10) with b^*, differentiate with respect to γ and evaluate the derivatives at $\gamma=1$ and $\Theta=0$ (noting that $\frac{d\Theta}{d\gamma} = -\beta$) to obtain

$$B_2\frac{\partial K}{\partial \gamma} = -E\{U''(\pi)pf_{K_1}(b-\beta)\ (pbf_{K_1}-r_K K-r(K)\ )\ K$$

$$+ U'(\pi)\ p(b-\beta)f_{K_1}(1+\varepsilon)\} \qquad (15)$$

Now for the risk neutral firm $U''(\pi) = 0$, therefore, the sign of (15) depends upon $-pE\{U'(\pi)\ (b-\beta)f_{K_1}(1+\varepsilon)\} = -p\mathrm{Cov}\{U'(\pi)f_{K_1}(1+\varepsilon),\ (b-\beta)\}$, which will be positive for the risk neutral firm provided the elasticity of marginal product curve is declining. Therefore, an increase in uncertainty will lead the risk neutral firm to hire less of the input.

The final case to be examined is the effect of an increase in the expected value of b. To proceed, replace b in (10) by $b^* = b+\Theta$ and differentiate with respect to Θ. Evaluating the derivative at $\Theta = 0$ then obtains

$$B_2\frac{\partial K}{\partial \Theta} = -E\{U''(\pi)\ pf_{K_1}K + (pbf_{K_1}-r(K)-r_K K) + U'(\pi)\ pf_{K_1}(1+\varepsilon)\} \qquad (16)$$

Our earlier assumptions about the elasticity of the marginal product curve implies that the term on the right hand side of (16) will be negative for the risk neutral firm. Therefore, as the expected flow of services obtainable from a given level of input hired rises the risk neutral monopsonist will hire more of the factor.

V. CONCLUDING REMARKS

It has been demonstrated that the standard results obtained for competitive firms in an uncertain world can be easily extended to the theory of monopsonistic firms. Namely, a monopsonistic firm facing price uncertainty or multiplicative production uncertainty will utilize less (more) of the input than it would in a certain world if the firm is risk averse (preferring). Further, an increase in uncertainty in these two cases will result in decreased factor utilization if the firm is characterized by nonincreasing absolute risk aversion.

For the risk neutral monopsonistic firm facing an uncertain flow of factor services it can be shown, under appropriate assumptions, that the firm will utilize less of the factor than it would in a world of certainty. Also, an increase in uncertainty will lead the risk-neutral firm to utilize less of the factor while an increase in the expected flow of factor services will prompt increased input hiring.

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FOOTNOTES

[1]Note that the inequality $U'(\pi)f_K(p-\mu) < U'\{E(\pi)\} f_K(p-\mu)$ holds for all p.

[2]As Ratti and Ullah (14) point out this restriction holds for the Cobb-Douglas, the CES and transcendental production functions. Of course, if this is not the case then the sign of $Cov(U'(\pi), bf_{K_1})$ will be the opposite of the sign of $U''(\pi)$. Somewhat paradoxically, this suggests that the risk averter will hire more than the risk preferring firm.

[3]It is interesting to note that Ratti and Ullah (4) use a similar argument, their equations 18-23, in an attempt to establish the same point for the competitive firm. While they establish the analogue of (14), their equation (23), they fail to realize that both sides of the analogue of (14) are equated to r_p in the competitive case which, of course, is a contradiction. It is not clear, therefore, that they have established a similar proposition for the competitive firm.

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The Monopsonistic Firm in an Uncertain World

In recent years the theory of the competitive firm operating in an uncertain environment has received a good deal of attention. Several excellent papers (Sandmo (5), Leland (3) and Batra and Ullah (2)) have examined the implications of an uncertain price for the competitive firm, while Ratti and Ullah (4) and Batra (1) have provided evidence on the competitive firm operating under production uncertainty.

Although Leland (3) has considered the behaviour of a price setting firm in an uncertain world, the case of a monopsonistic firm has apparently not yet been examined. The purpose of this note is to demonstrate that the results obtained for competitive firms are, in the main, easily extended to the case of the monopsonistic firm. To sharpen the results and to simplify the derivations a simple model requiring only one input is used.

I. THE MODEL AND ASSUMPTIONS

Three different types of uncertainty will be examined. Case 1 considers the monopsonistic firm facing an uncertain price p with density function f(p) and mean $E(p) = \mu$. Production is characterized by $q = f(k)$ where q is output and K is input; it is assumed that $f_K > 0$ and $f_{KK} < 0$. Case 2 examines the behaviour of the monopsonistic firm under production uncertainty characterized by the production relationship $q = af(K)$ where a is a random variable with mean $E(a) = \alpha$ and density f(a). In this case p is non-stochastic and it is assumed that $f_K > 0$ and $f_{KK} < 0$. The final case to be considered is that of production uncertainty generated by the production relationship $q = f(bK)$ where